8

Cavitation in Hydraulic Machinery

8.1 INTRODUCTION

Cavitation is caused by local vaporization of the fluid, when the local static pressure of a liquid falls below the vapor pressure of the liquid. Small bubbles or cavities filled with vapor are formed, which suddenly collapse on moving forward with the flow into regions of high pressure. These bubbles collapse with tremendous force, giving rise to as high a pressure as 3500 atm. In a centrifugal pump, these low-pressure zones are generally at the impeller inlet, where the fluid is locally accelerated over the vane surfaces. In turbines, cavitation is most likely to occur at the downstream outlet end of a blade on the low-pressure leading face. When cavitation occurs, it causes the following undesirable effects:

1. Local pitting of the impeller and erosion of the metal surface.
2. Serious damage can occur from prolonged cavitation erosion.
3. Vibration of machine; noise is also generated in the form of sharp cracking sounds when cavitation takes place.
4. A drop in efficiency due to vapor formation, which reduces the effective flow areas.

The avoidance of cavitation in conventionally designed machines can be regarded as one of the essential tasks of both pump and turbine designers. This cavitation imposes limitations on the rate of discharge and speed of rotation of the pump.

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8.2 STAGES AND TYPES OF CAVITATION

The term *incipient stage* describes cavitation that is just barely detectable. The discernible bubbles of incipient cavitation are small, and the zone over which cavitation occurs is limited. With changes in conditions (pressure, velocity, temperature) toward promoting increased vaporization rates, cavitation grows; the succeeding stages are distinguished from the incipient stage by the term *developed*.

Traveling cavitation is a type composed of individual transient cavities or bubbles, which form in the liquid, as they expand, shrink, and then collapse. Such traveling transient bubbles may appear at the low-pressure points along a solid boundary or in the liquid interior either at the cores of moving vortices or in the high-turbulence region in a turbulent shear field.

The term *fixed cavitation* refers to the situation that sometimes develops after inception, in which the liquid flow detaches from the rigid boundary of an immersed body or a flow passage to form a pocket or cavity attached to the boundary. The attached or fixed cavity is stable in a quasi-steady sense. Fixed cavities sometimes have the appearance of a highly turbulent boiling surface.

In *vortex cavitation*, the cavities are found in the cores of vortices that form in zones of high shear. The cavitation may appear as traveling cavities or as a fixed cavity. Vortex cavitation is one of the earliest observed types, as it often occurs on the blade tips of ships' propellers. In fact, this type of cavitation is often referred to as "tip" cavitation. Tip cavitation occurs not only in open propellers but also in ducted propellers such as those found in propeller pumps at hydrofoil tips.

8.2.1 Cavitation on Moving Bodies

There is no essential difference between cavitation in a flowing stream and that on a body moving through a stationary liquid. In both cases, the important factors are the relative velocities and the absolute pressures. When these are similar, the same types of cavitation are found. One noticeable difference is that the turbulence level in the stationary liquid is lower. Many cases of cavitation in a flowing stream occur in relatively long flow passages in which the turbulence is fully developed before the liquid reaches the cavitation zone. Hydraulic machinery furnishes a typical example of a combination of the two conditions. In the casing, the moving liquid flows past stationary guide surfaces; in the runner, the liquid and the guide surfaces are both in motion.

8.2.2 Cavitation Without Major Flow—Vibratory Cavitation

The types of cavitation previously described have one major feature in common. It is that a particular liquid element passes through the cavitation zone only once. *Vibratory cavitation* is another important type of cavitation, which does not have

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this characteristic. Although it is accompanied sometimes by continuous flow, the velocity is so low that a given element of liquid is exposed to many cycles of cavitation (in a time period of the order of milliseconds) rather than only one. In vibratory cavitation, the forces causing the cavities to form and collapse are due to a continuous series of high-amplitude, high-frequency pressure pulsations in the liquid. These pressure pulsations are generated by a submerged surface, which vibrates normal to its face and sets up pressure waves in the liquid. No cavities will be formed unless the amplitude of the pressure variation is great enough to cause the pressure to drop to or below the vapor pressure of the liquid. As the vibratory pressure field is characteristic of this type of cavitation, the name "vibratory cavitation" follows.

8.3 EFFECTS AND IMPORTANCE OF CAVITATION

Cavitation is important as a consequence of its effects. These may be classified into three general categories:

1. Effects that modify the hydrodynamics of the flow of the liquid
2. Effects that produce damage on the solid-boundary surfaces of the flow
3. Extraneous effects that may or may not be accompanied by significant hydrodynamic flow modifications or damage to solid boundaries

Unfortunately for the field of applied hydrodynamics, the effects of cavitation, with very few exceptions, are undesirable. Uncontrolled cavitation can produce serious and even catastrophic results. The necessity of avoiding or controlling cavitation imposes serious limitations on the design of many types of hydraulic equipment. The simple enumeration of some types of equipment, structures, or flow systems, whose performance may be seriously affected by the presence of cavitation, will serve to emphasize the wide occurrence and the relative importance of this phenomenon.

In the field of hydraulic machinery, it has been found that all types of turbines, which form a low-specific-speed Francis to the high-specific-speed Kaplan, are susceptible to cavitation. Centrifugal and axial-flow pumps suffer from its effects, and even the various types of positive-displacement pumps may be troubled by it. Although cavitation may be aggravated by poor design, it may occur in even the best-designed equipment when the latter is operated under unfavorable condition.

8.4 CAVITATION PARAMETER FOR DYNAMIC SIMILARITY

The main variables that affect the inception and subsequent character of cavitation in flowing liquids are the boundary geometry, the flow variables

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of absolute pressure and velocity, and the critical pressure p_{crit} at which a bubble can be formed or a cavity maintained. Other variables may cause significant variations in the relation between geometry, pressure, and velocity and in the value of the critical pressure. These include the properties of the liquid (such as viscosity, surface tension, and vaporization characteristics), any solid, or gaseous contaminants that may be entrained or dissolved in the liquid, and the condition of the boundary surfaces, including cleanliness and existence of crevices, which might host undissolved gases. In addition to dynamic effects, the pressure gradients due to gravity are important for large cavities whether they be traveling or attached types. Finally, the physical size of the boundary geometry may be important, not only in affecting cavity dimensions but also in modifying the effects of some of the fluid and boundary flow properties.

Let us consider a simple liquid having constant properties and develop the basic cavitation parameter. A relative flow between an immersed object and the surrounding liquid results in a variation in pressure at a point on the object, and the pressure in the undisturbed liquid at some distance from the object is proportional to the square of the relative velocity. This can be written as the negative of the usual pressure coefficient C_{p}, namely,

$$-C_{\mathrm{p}} = \frac{(p_0 - p)_{\mathrm{d}}}{\rho V_0^2/2} \tag{8.1}$$

where ρ is the density of liquid, V_0 the velocity of undisturbed liquid relative to body, p_0 the pressure of undisturbed liquid, p the pressure at a point on object, and $(p_0 - p)_{\mathrm{d}}$ the pressure differential due to *dynamic effects* of fluid motion.

This is equivalent to omitting gravity. However, when necessary, gravity effects can be included.

At some location on the object, p will be a minimum, p_{min}, so that

$$(-C_{\mathrm{p}})_{\mathrm{min}} = \frac{p_0 - p_{\mathrm{min}}}{\rho V_0^2/2} \tag{8.2}$$

In the absence of cavitation (and if Reynolds-number effects are neglected), this value will depend only on the shape of the object. It is easy to create a set of conditions such that p_{min} drops to some value at which cavitation exists. This can be accomplished by increasing the relative velocity V_0 for a fixed value of the pressure p_0 or by continuously lowering p_0 with V_0 held constant. Either procedure will result in lowering of the absolute values of all the local pressures on the surface of the object. If surface tension is ignored, the pressure p_{min} will be the pressure of the contents of the cavitation cavity. Denoting this as a bubble pressure p_{b}, we can define a *cavitation parameter* by replacing p_{min}; thus

$$K_{\mathrm{b}} = \frac{p_0 - p_{\mathrm{b}}}{\rho V_0^2/2} \tag{8.3}$$

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or, in terms of pressure head (in feet of the liquid),

$$K_b = \frac{(p_0 - p_b)/\gamma}{V_0^2/2g} \tag{8.4}$$

where p_0 is the absolute-static pressure at some reference locality, V_0 the reference velocity, p_b the absolute pressure in cavity or bubble, and γ the specific weight of liquid.

If we now assume that cavitation will occur when the normal stresses at a point in the liquid are reduced to zero, p_b will equal the vapor pressure p_v.

Then, we write

$$K_b = \frac{p_0 - p_v}{\rho V_0^2/2} \tag{8.5}$$

The value of K at which cavitation inception occurs is designated as K_i. A theoretical value of K_i is the magnitude $|(-C_p)_{min}|$ for any particular body.

The initiation of cavitation by vaporization of the liquid may require that a negative stress exist because of surface tension and other effects. However, the presence of such things as undissolved gas particles, boundary layers, and turbulence will modify and often mask a departure of the critical pressure p_{crit} from p_v. As a consequence, Eq. (8.5) has been universally adopted as the parameter for comparison of vaporous cavitation events.

The beginning of cavitation means the appearance of tiny cavities at or near the place on the object where the minimum pressure is obtained. Continual increase in V_0 (or decrease in p_0) means that the pressure at other points along the surface of the object will drop to the critical pressure. Thus, the zone of cavitation will spread from the location of original inception. In considering the behavior of the cavitation parameter during this process, we again note that if Reynolds-number effects are neglected the pressure coefficient $(-C_p)_{min}$ depends only on the object's shape and is constant prior to inception. After inception, the value decreases as p_{min} becomes the cavity pressure, which tends to remain constant, whereas either V_0 increases or p_0 decreases. Thus, the cavitation parameter assumes a definite value at each stage of development or "degree" of cavitation on a particular body. For inception, $K = K_i$; for advanced stages of cavitation, $K < K_i$. K_i and values of K at subsequent stages of cavitation depend primarily on the shape of the immersed object past which the liquid flows.

We should note here that for flow past immersed objects and curved boundaries, K_i will always be finite. For the limiting case of parallel flow of an ideal fluid, K_i will be zero since the pressure p_0 in the main stream will be the same as the wall pressure (again with gravity omitted and the assumption that cavitation occurs when the normal stresses are zero).

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8.4.1 The Cavitation Parameter as a Flow Index

The parameter K_b or K can be used to relate the conditions of flow to the possibility of cavitation occurring as well as to the degree of postinception stages of cavitation. For any system where the existing or potential bubble pressure (p_b or p_v) is fixed, the parameter (K_b or K) can be computed for the full range of values of the reference velocity V_0 and reference pressure p_0. On the other hand, as previously noted, for any degree of cavitation from inception to advanced stages, the parameter has a characteristic value. By adjusting the flow conditions so that K is greater than, equal to, or less than K_i, the full range of possibilities, from no cavitation to advanced stages of cavitation, can be established.

8.4.2 The Cavitation Parameter in Gravity Fields

As the pressure differences in the preceding relations are due to dynamic effects, the cavitation parameter is defined independently of the gravity field. For large bodies going through changes in elevation as they move, the relation between dynamic pressure difference $(p_0 - p_{\min})_d$ and the actual pressure difference $(p_0 - p_{\min})_{\text{actual}}$ is

$$(p_0 - p_{\min})_d = (p_0 - p_{\min})_{\text{actual}} + \gamma(h_0 - h_{\min})$$

where γ is the liquid's specific weight and h is elevation. Then, in terms of actual pressures, we have, instead of Eq. (8.5),

$$K = \frac{(p_0 + \gamma h_0) - (p_v + \gamma h_{\min})}{\rho V_0/2} \tag{8.6}$$

For $h_0 = h_{\min}$, Eq. (8.6) reduces to Eq. (8.5).

8.5 PHYSICAL SIGNIFICANCE AND USES OF THE CAVITATION PARAMETER

A simple physical interpretation follows directly when we consider a cavitation cavity that is being formed and then swept from a low-pressure to a high-pressure region. Then the numerator is related to the net pressure or head, which tends to collapse the cavity. The denominator is the velocity pressure or head of the flow. The variations in pressure, which take place on the surface of the body or on any type of guide passage, are basically due to changes in the velocity of the flow. Thus, the velocity head may be considered to be a measure of the pressure reductions that may occur to cause a cavity to form or expand.

The basic importance of cavitation parameter stems from the fact that it is an index of dynamic similarity of flow conditions under which cavitation occurs. Its use, therefore, is subject to a number of limitations. Full dynamic similarity

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between flows in two systems requires that the effects of all physical conditions be reproduced according to unique relations. Thus, even if identical thermodynamics and chemical properties and identical boundary geometry are assumed, the variable effects of contaminants in the liquid-omitted dynamic similarity require that the effects of viscosity, gravity, and surface tension be in unique relationship at each cavitation condition. In other words, a particular cavitation condition is accurately reproduced only if Reynolds number, Froude number, Weber number, etc. as well as the cavitation parameter K have particular values according to a unique relation among themselves.

8.6 THE RAYLEIGH ANALYSIS OF A SPHERICAL CAVITY IN AN INVISCID INCOMPRESSIBLE LIQUID AT REST AT INFINITY

The mathematical analysis of the formation and collapse of spherical cavities, which are the idealized form of the traveling transient cavities, has proved interesting to many workers in the field. Furthermore, it appears that as more experimental evidence is obtained on the detailed mechanics of the cavitation process, the role played by traveling cavities grows in importance. This is especially true with respect to the process by which cavitation produces physical damage on the guiding surfaces.

Rayleigh first set up an expression for the velocity u, at any radial distance r, where r is greater than R, the radius of the cavity wall. U is the cavity-wall velocity at time t. For spherical symmetry, the radial flow is irrotational with velocity potential, and velocity is given by

$$\phi = \frac{UR^2}{r} \quad \text{and} \quad \frac{u}{U} = \frac{R^2}{r^2} \tag{8.7}$$

Next, the expression for the kinetic energy of the entire body of liquid at time t is developed by integrating kinetic energy of a concentric fluid shell of thickness dr and density ρ. The result is

$$(\mathrm{KE})_{\mathrm{liq}} = \frac{\rho}{2}\int_R^{\infty} u^2 4\pi r^2 \mathrm{d}r = 2\pi\rho U^2 R^3 \tag{8.8}$$

The work done on the entire body of fluid as the cavity is collapsing from the initial radius R_0 to R is a product of the pressure p_∞ at infinity and the change in volume of the cavity as no work is done at the cavity wall where the pressure is assumed to be zero, i.e.,

$$\frac{4\pi p_\infty}{3}\left(R_0^3 - R^3\right) \tag{8.9}$$

If the fluid is inviscid as well as incompressible, the work done appears as kinetic

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energy. Therefore, Eq. (8.8) can be equated to Eq. (8.9), which gives

$$U^2 = \frac{2p_\infty}{3\rho}\left(\frac{R_0^3}{R^3} - 1\right) \tag{8.10}$$

An expression for the time t required for a cavity to collapse from R_0 to R can be obtained from Eq. (8.10) by substituting for the velocity U of the boundary, its equivalent dR/dt and performing the necessary integration.

This gives

$$t = \sqrt{\frac{3\rho}{2p_\infty}}\int_R^{R_0} \frac{R^{3/2}\mathrm{d}R}{\left(R_0^3 - R^3\right)^{1/2}} = R_0\sqrt{\frac{3\rho}{2p_\infty}}\int_\beta^1 \frac{\beta^{3/2}\mathrm{d}\beta}{(1-\beta^3)^{1/2}} \tag{8.11}$$

The new symbol β is R/R_0. The time τ of complete collapse is obtained if Eq. (8.11) is evaluated for $\beta = 0$. For this special case, the integration may be performed by means of functions with the result that τ becomes

$$\tau = R_0\sqrt{\frac{\rho}{6p_\infty}} \times \frac{\Gamma\left(\frac{3}{6}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{4}{3}\right)} = 0.91468R_0\sqrt{\frac{\rho}{p_\infty}} \tag{8.12}$$

Rayleigh did not integrate Eq. (8.11) for any other value of β. In the detailed study of the time history of the collapse of a cavitation bubble, it is convenient to have a solution for all values of β between 0 and 1.0. Table 8.1 gives values of the dimensionless time $t = t/R_0\sqrt{\rho/p_\infty}$ over this range as obtained from a numerical solution of a power series expansion of the integral in Eq. (8.11).

Equation (8.10) shows that as R decreases to 0, the velocity U increases to infinity. In order to avoid this, Rayleigh calculated what would happen if, instead of having zero or constant pressure within the cavity, the cavity is filled with a gas, which is compressed isothermally. In such a case, the external work done on the system as given by Eq. (8.9) is equated to the sum of the kinetic energy of the liquid given by Eq. (8.8) and the work of compression of the gas, which is $4\pi Q R_0^3 \ln(R_0/R)$, where Q is the initial pressure of the gas. Thus Eq. (8.10) is replaced by

$$U^2 = \frac{2p_\infty}{3\rho}\left(\frac{R_0^3}{R^3} - 1\right) - \frac{2Q}{\rho} \times \frac{R_0^3}{R^3}\ln_0\frac{R_0}{R} \tag{8.13}$$

For any real (i.e., positive) value of Q, the cavity will not collapse completely, but U will come to 0 for a finite value of R. If Q is greater than p_∞, the first movement of the boundary is outward. The limiting size of the cavity can be obtained by setting $U = 0$ in Eq. (8.13), which gives

$$p_\infty \frac{z-1}{z} - Q\ln z = 0 \tag{8.14}$$

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Table 8.1 Values of the Dimensionless Time $t' = t/R_0\sqrt{\rho/p_\infty}$ from Eq. (8.11) (Error Less Than 10^{-6} for $0 \le \beta \le 0.96$)

β	$t\frac{\sqrt{p_\infty/\rho}}{R_0}$	β	$t\frac{\sqrt{p_\infty/\rho}}{R_0}$	β	$t\frac{\sqrt{p_\infty/\rho}}{R_0}$
0.99	0.016145	0.64	0.733436	0.29	0.892245
0.98	0.079522	0.63	0.741436	0.28	0.894153
0.97	0.130400	0.62	0.749154	0.27	0.895956
0.96	0.174063	0.61	0.756599	0.26	0.897658
0.95	0.212764	0.60	0.763782	0.25	0.899262
0.94	0.247733	0.59	0.770712	0.24	0.900769
0.93	0.279736	0.58	0.777398	0.23	0.902182
0.92	0.309297	0.57	0.783847	0.22	0.903505
0.91	0.336793	0.56	0.790068	0.21	0.904738
0.90	0.362507	0.55	0.796068	0.20	0.905885
0.89	0.386662	0.54	0.801854	0.19	0.906947
0.88	0.409433	0.53	0.807433	0.18	0.907928
0.87	0.430965	0.52	0.812810	0.17	0.908829
0.86	0.451377	0.51	0.817993	0.16	0.909654
0.85	0.470770	0.50	0.822988	0.15	0.910404
0.84	0.489229	0.49	0.827798	0.14	0.911083
0.83	0.506830	0.48	0.832431	0.13	0.911692
0.82	0.523635	0.47	0.836890	0.12	0.912234
0.81	0.539701	0.46	0.841181	0.11	0.912713
0.80	0.555078	0.45	0.845308	0.10	0.913130
0.79	0.569810	0.44	0.849277	0.09	0.913489
0.78	0.583937	0.43	0.853090	0.08	0.913793
0.77	0.597495	0.42	0.856752	0.07	0.914045
0.76	0.610515	0.41	0.860268	0.06	0.914248
0.75	0.623027	0.40	0.863640	0.05	0.914406
0.74	0.635059	0.39	0.866872	0.04	0.914523
0.73	0.646633	0.38	0.869969	0.03	0.914604
0.72	0.657773	0.37	0.872933	0.02	0.914652
0.71	0.668498	0.36	0.875768	0.01	0.914675
0.70	0.678830	0.35	0.878477	0.00	0.914680
0.69	0.688784	0.34	0.887062		
0.68	0.698377	0.33	0.883528		
0.67	0.707625	0.32	0.885876		
0.66	0.716542	0.31	0.222110		
0.65	0.725142	0.30	0.890232		

in which z denotes the ratio of the volume R_0^3/R^3. Equation (8.14) indicates that the radius oscillates between the initial value R_0 and another, which is determined by the ratio p_∞/Q from this equation. If $p_\infty/Q > 1$, the limiting size is a minimum. Although Rayleigh presented this example only for isothermal

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compression, it is obvious that any other thermodynamic process may be assumed for the gas in the cavity, and equations analogous to Eq. (8.13) may be formulated.

As another interesting aspect of the bubble collapse, Rayleigh calculated the pressure field in the liquid surrounding the bubble reverting to the empty cavity of zero pressure. He set up the radial acceleration as the total differential of the liquid velocity u, at radius r, with respect to time, equated this to the radial pressure gradient, and integrated to get the pressure at any point in the liquid. Hence,

$$a_r = -\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{\partial u}{\partial t} - u\frac{\partial u}{\partial t} = \frac{1}{\rho}\frac{\partial p}{\partial r} \tag{8.15}$$

Expressions for $\partial u/\partial t$ and $u(\partial u/\partial r)$ as functions of R and r are obtained from Eqs. (8.7) and (8.10), the partial differential of Eq. (8.7) being taken with respect to r and t, and the partial differential of Eq. (8.7) with respect to t. Substituting these expressions in Eq. (8.15) yields:

$$\frac{1}{p_\infty}\frac{\partial p}{\partial r} = \frac{R}{3r^2}\left[\frac{(4z-4)R^3}{r^3} - (z-4)\right] \tag{8.16}$$

in which $z = *(R_0/R)^3$ and $r \leq R$ always. By integration, this becomes

$$\frac{1}{p_\infty}\int_{p_\infty}^{p} \mathrm{d}p = \frac{R}{3}\left[(4z-4)R^3\int_{\infty}^{r}\frac{\mathrm{d}r}{r^5} - (z-4)\int_{\infty}^{r}\frac{\mathrm{d}r}{r^2}\right] \tag{8.17}$$

which gives

$$\frac{p}{p_\infty} - 1 = \frac{R}{3r}(z-4) - \frac{R^4}{3r^4}(z-1) \tag{8.18}$$

The pressure distribution in the liquid at the instant of release is obtained by substituting $R = R_0$ in Eq. (8.18), which gives

$$p = p_\infty\left(1 - \frac{R_0}{r}\right) \tag{8.19}$$

In Eq. (8.18), $z = 1$ at the initiation of the collapse and increases as collapse proceeds. Figure 8.1 shows the distribution of the pressure in the liquid according to Eq. (8.18). It is seen that for $1 < z < 4$, $p_{\mathrm{max}} = p_\infty$ and occurs at $R/r = 0$, where $r \to \infty$. For $4 < z < \infty$, $p_{\mathrm{max}} > p_\infty$ and occurs at finite r/R. This location moves toward the bubble with increasing z and approaches $r/R = 1.59$ as z approaches infinity. The location r_{m} of the maximum pressure in the liquid may be found by setting $\mathrm{d}p/\mathrm{d}r$ equal to zero in Eq. (8.16). This gives a maximum value for p when

$$\frac{r_{\mathrm{m}}^3}{R^3} = \frac{4z-4}{z-4} \tag{8.20}$$

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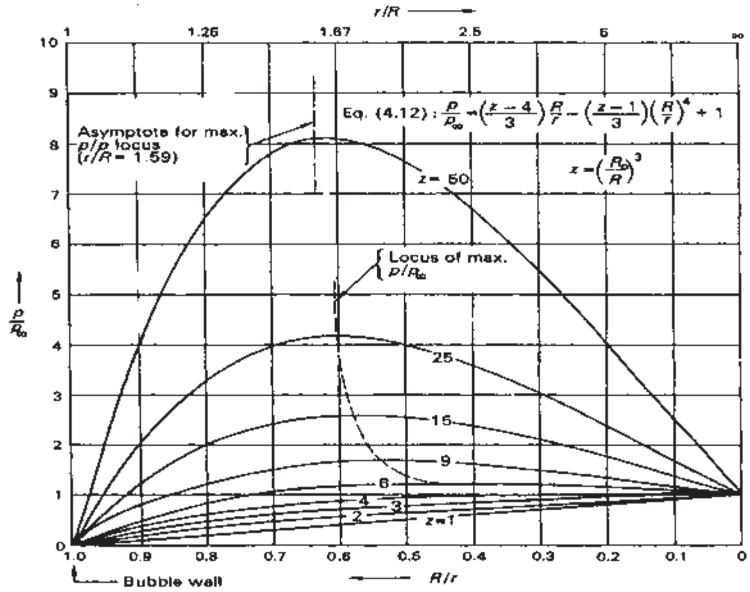

Figure 8.1 Rayleigh analysis: pressure profile near a collapsing bubble.

When r_m is substituted for r in Eq. (8.18), the maximum value of p is obtained as

$$\frac{p_{max}}{p_\infty} = 1 + \frac{(z-4)R}{4r_m} = 1 + \frac{(z-4)^{4/3}}{4^{4/3}(z-1)^{1/3}} \tag{8.21}$$

As cavity approaches complete collapse, z becomes great, and Eqs. (8.20) and (8.21) may be approximated by

$$r_m = 4^{1/3}R = 1.587R \tag{8.22}$$

and

$$\frac{p_{max}}{p_\infty} = \frac{z}{4^{4/3}} = \frac{R_0^3}{4^{4/3}R^3} \tag{8.23}$$

Equations (8.22) and (8.23) taken together show that as the cavity becomes very small, the pressure in the liquid near the boundary becomes very great in spite of the fact that the pressure at the boundary is always zero. This would suggest the possibility that in compressing the liquid some energy can be stored, which would add an additional term to Eq. (8.10). This would invalidate the assumption of incompressibility. Rayleigh himself abandoned this assumption in considering what happens if the cavity collapses on an absolute rigid sphere of radius R. In this treatment, the assumption of incompressibility is abandoned only at

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the instant that the cavity wall comes in contact with the rigid sphere. From that instant, it is assumed that the kinetic energy of deformation of the same particle is determined by the bulk modulus of elasticity of the fluid, as is common in water-hammer calculations. On this basis, it is found that

$$\frac{(P')^2}{2E}\frac{1}{2} = \rho U^2 = \frac{p_\infty}{3}\left(\frac{R_0^3}{R^3} - 1\right) = \frac{p_\infty}{3}(z - 1) \tag{8.24}$$

where P' is the instantaneous pressure on the surface of the rigid sphere and E is the bulk modulus of elasticity. Both must be expressed in the same units.

It is instructive to compare the collapse of the cavity with the predicted collapse based on this simple theory. Figure 8.2 shows this comparison.

This similarity is very striking, especially when it is remembered that there was some variation of pressure p_∞ during collapse of the actual cavity. It will be noted that the actual collapse time is greater than that predicted by Eq. (8.12).

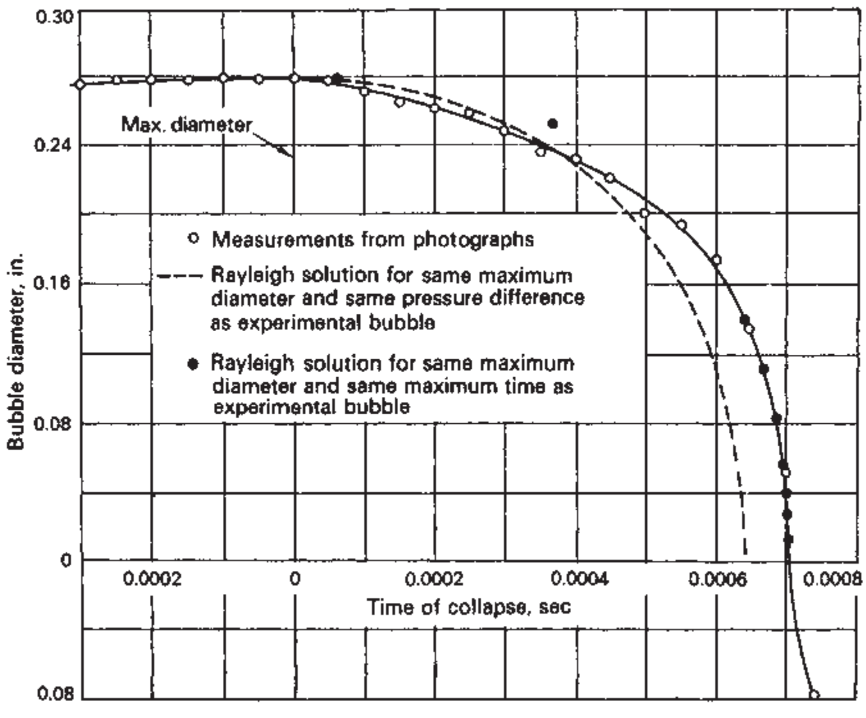

Figure 8.2 Comparison of measured bubble size with the Rayleigh solution for an empty cavity in an incompressible liquid with a constant pressure field.

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8.7 CAVITATION EFFECTS ON PERFORMANCE OF HYDRAULIC MACHINES

8.7.1 Basic Nature of Cavitation Effects on Performance

The effects of cavitation on hydraulic performance are many and varied. They depend upon the type of equipment or structure under consideration and the purpose it is designed to fulfill. However, the basic elements, which together make up these effects on performance, are stated as follows:

1. The presence of a cavitation zone can change the friction losses in a fluid flow system, both by altering the skin friction and by varying the form resistance. In general, the effect is to increase the resistance, although this is not always true.
2. The presence of a cavitation zone may result in a change in the local direction of the flow due to a change in the lateral force, which a given element of guiding surface can exert on the flow as it becomes covered by cavitation.
3. With well-developed cavitation the decrease in the effective cross-section of the liquid-flow passages may become great enough to cause partial or complete breakdown of the normal flow.

The development of cavitation may seriously affect the operation of all types of hydraulic structures and machines. For example, it may change the rate of discharge of gates or spillways, or it may lead to undesirable or destructive pulsating flows. It may distort the action of control valves and other similar flow devices. However, the most trouble from cavitation effects has been experienced in rotating machinery; hence, more is known about the details of these manifestations. Study of these details not only leads to a better understanding of the phenomenon in this class of equipment but also sheds considerable light on the reason behind the observed effects of cavitation in many types of equipment for which no such studies have been made. Figures 8.3 and 8.4 display the occurrence of cavitation and its effect on the performance of a centrifugal pump.

8.8 THOMA'S SIGMA AND CAVITATION TESTS

8.8.1 Thoma's Sigma

Early in the study of the effects of cavitation on performance of hydraulic machines, a need developed for a satisfactory way of defining the operating conditions with respect to cavitation. For example, for the same machine operating under different heads and at different speeds, it was found desirable

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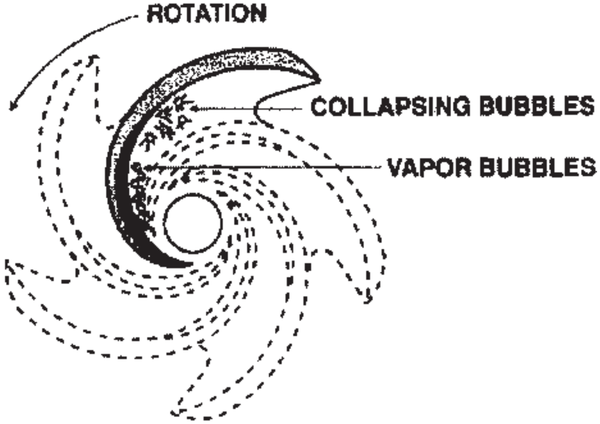

Figure 8.3 Cavitation occurs when vapor bubbles form and then subsequently collapse as they move along the flow path on an impeller.

to specify the conditions under which the degree of cavitation would be similar. It is sometimes necessary to specify similarity of cavitation conditions between two machines of the same design but of different sizes, e.g., between model and prototype. The cavitation parameter commonly accepted for this purpose was

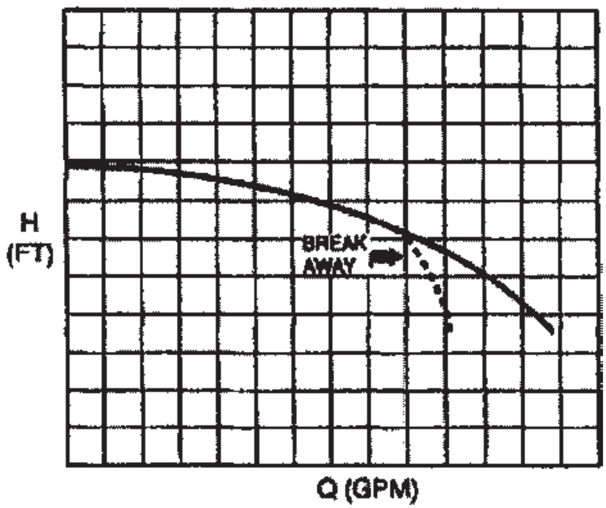

Figure 8.4 Effect of cavitation on the performance of a centrifugal pump.

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proposed by Thoma and is now commonly known as the Thoma sigma, σ_T. For general use with pumps or turbines, we define sigma as

$$\sigma_{sv} = \frac{H_{sv}}{H} \tag{8.25}$$

where H_{sv}, the *net positive suction head* at some location = total absolute head less vapor-pressure head = $[(p_{atm}/\gamma) + (p/\gamma) + (V^2/2g) - (p_v/\gamma)]$, H is the head produced (pump) or absorbed (turbine), and γ is the specific weight of fluid.

For turbines with negative static head on the runner,

$$H_{sv} = H_a - H_s - H_v + \frac{V_e^2}{2g} + H_f \tag{8.26}$$

where H_a is the barometric-pressure head, H_s the static draft head defined as elevation of runner discharge above surface of tail water, H_v the vapor-pressure head, V_e the draft-tube exit average velocity (tailrace velocity), and H_f the draft-tube friction loss.

If we neglect the draft-tube friction loss and exit-velocity head, we get sigma in Thoma's original form:

$$\sigma_T = \frac{H_a - H_s - H_v}{H} \tag{8.27}$$

Thus

$$\sigma_T = \sigma_{sv} - \frac{V_e^2/2g + H_f}{H} \tag{8.28}$$

Sigma (σ_{sv} or σ_T) has a definite value for each installation, known as the plant sigma. Every machine will cavitate at some critical sigma (σ_{sv_c} or σ_{T_c}). Clearly, cavitation will be avoided only if the plant sigma is greater than the critical sigma.

The cavitation parameter for the flow passage at the turbine runner discharge is, say,

$$K_d = \frac{H_d - H_v}{V_d^2/2g} \tag{8.29}$$

where H_d is the absolute-pressure head at the runner discharge and V_d the average velocity at the runner discharge. Equation (8.29) is similar in form to Eq. (8.25) but they do not have exactly the same significance. The numerator of K_d is the actual cavitation-suppression pressure head of the liquid as it discharges from the runner. (This assumes the same pressure to exist at the critical location for cavitation inception.) Its relation to the numerator of σ_T is

$$H_d - H_v = H_{sv} - \frac{V_d^2}{2g} \tag{8.30}$$

For a particular machine operating at a particular combination of the operating variables, flow rate, head speed, and wicket-gate setting,

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$$\frac{V_d^2}{2g} = C_1 H \tag{8.31}$$

Using the previous relations, it can be shown that Eq. (8.29) may be written as

$$K_d = \frac{\sigma_T}{C_1} - \left(1 - \frac{H_f}{C_1 H}\right) + \frac{V_e^2/2g}{V_d^2/2g}$$

The term in parenthesis is the efficiency of the draft tube, η_{dt}, as the converter of the entering velocity head to pressure head. Thus the final expression is

$$K_d = \frac{\sigma_T}{C_1} - \eta_{dt} + \frac{V_e^2}{V_d^2} \tag{8.32}$$

C_1 is a function of both design of the machine and the setting of the guide vane; η_{dt} is a function of the design of the draft tube but is also affected by the guide-vane setting. If a given machine is tested at constant guide-vane setting and operating specific speed, both C_1 and η_{dt} tend to be constant; hence σ_T and K_d have a linear relationship. However, different designs usually have different values of C_1 even for the same specific speed and vane setting, and certainly for different specific speeds. K_d, however, is a direct measure of the tendency of the flow to produce cavitation, so that if two different machines of different designs cavitated at the same value of K_d it would mean that their guiding surfaces in this region had the dame value of K_i. However, sigma values could be quite different. From this point of view, sigma is not a satisfactory parameter for the comparison of machines of different designs. On the other hand, although the determination of the value of K_d for which cavitation is incipient is a good measure of the excellence of the *shape* of the passages in the discharge region, it sheds no light on whether or not the cross-section is an optimum as well. In this respect, sigma is more informative as it characterizes the discharge conditions by the total head rather than the velocity head alone.

Both K_d and sigma implicitly contain one assumption, which should be borne in mind because at times it may be rather misleading. The critical cavitation zone of the turbine runner is in the discharge passage just downstream from the turbine runner. Although this is usually the minimum-pressure point in the system, it is not necessarily the cross-section that may limit the cavitation performance of the machine. The critical cross-section may occur further upstream on the runner blades and may frequently be at the entering edges rather than trailing edges. However, these very real limitations and differences do not alter the fact that K_d and σ_T are both cavitation parameters and in many respects, they can be used in the same manner. Thus K_d (or K evaluated at any location in the machine) can be used to measure the tendency of the flow to cavitate, the conditions of the flow at which cavitation first begins (K_i), or the conditions of the flow corresponding to a certain degree of development of cavitation.

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Likewise, σ_T can be used to characterize the tendency of the flow through a machine to cause cavitation, the point of inception of cavitation, the point at which cavitation first affects the performance, or the conditions for complete breakdown of performance.

K_i is a very general figure of merit, as its numerical value gives directly the resistance of a given guiding surface to the development of cavitation. Thoma's sigma can serve the same purpose for the entire machine, but in a much more limited sense. Thus, for example, σ_T can be used directly to compare the cavitation resistance of a series of different machines, all designed to operate under the same total head. However, the numerical value of σ_T, which characterizes a very good machine, for one given head may indicate completely unacceptable performance for another. Naturally, there have been empirical relations developed through experience, which show how the σ_T for acceptable performance varies with the design conditions. Figure 8.5 shows such a relationship.

Here, the specific speed has been taken as the characteristic that describes the design type. It is defined for turbines as

$$N_s = \frac{N\sqrt{\text{hp}}}{H^{5/4}} \tag{8.33}$$

where N is the rotating speed, hp the power output, and H the turbine head.

The ordinate is plant sigma ($\sigma_T = \sigma_{\text{plant}}$). Both sigma and specific speed are based on rated capacity at the design head.

In the use of such diagrams, it is always necessary to understand clearly the basis for their construction. Thus, in Fig. 8.5, the solid lines show the minimum-plant sigma for each specific speed at which a turbine can reasonably be expected to perform satisfactorily; i.e., cavitation will be absent or so limited as not to cause efficiency loss, output loss, undesirable vibration, unstable flow, or excessive pitting. Another criterion of satisfactory operation might be that cavitation damage should not exceed a specific amount, measured in pounds of metal removed per year. Different bases may be established to meet other needs. A sigma curve might be related to hydraulic performance by showing the limits of operation for a given drop in efficiency or for a specific loss in power output.

Although the parameter sigma was developed to characterize the performance of hydraulic turbines, it is equally useful with pumps. For pumps, it is used in the form of Eq. (8.25). In current practice, the evaluation of H_{sv} varies slightly depending on whether the pump is supplied directly from a forebay with a free surface or forms a part of a closed system. In the former case, H_{sv} is calculated by neglecting forebay velocity and the friction loss between the forebay and the inlet, just as the tailrace velocity and friction loss between the turbine-runner discharge and tail water are neglected. In the latter case, H_{sv} is calculated from the pressure measured at the inlet. Velocity is assumed to be the average velocity, Q/A. Because of this difference in meaning, if the same

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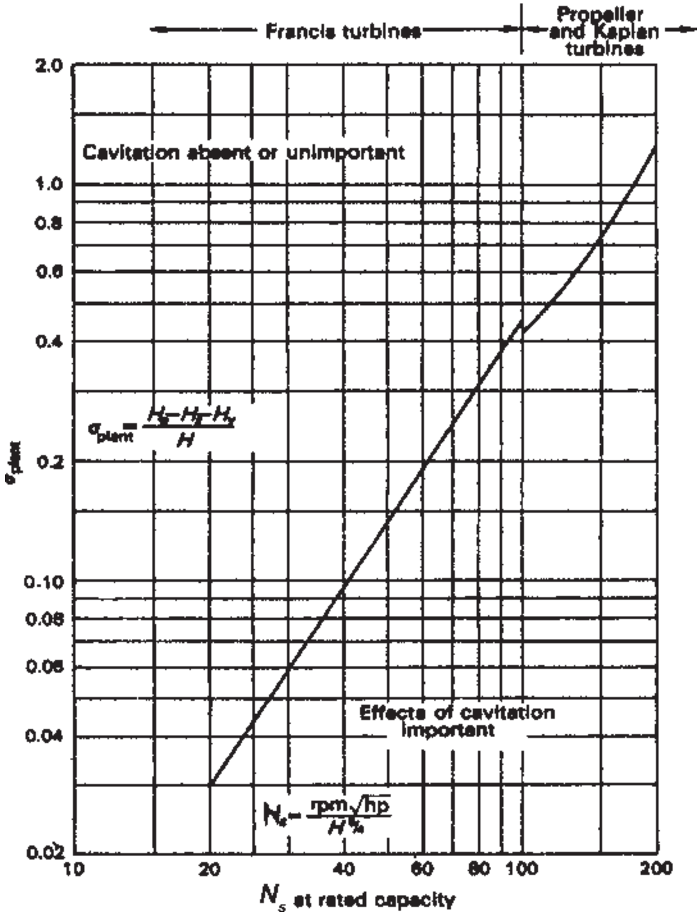

Figure 8.5 Experience limits of plant sigma vs. specific speed for hydraulic turbines.

machine was tested under both types of installation, the results would apparently show a slightly poor cavitation performance with the forebay.

8.8.2 Sigma Tests

Most of the detailed knowledge of the effect of cavitation on the performance of hydraulic machines has been obtained in the laboratory, because of the difficulty encountered in nearly all field installations in varying the operating conditions over a wide enough range. In the laboratory, the normal procedure is to obtain data for the plotting of a group of σ_T curves. Turbine cavitation tests are best

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run by operating the machine at fixed values of turbine head, speed, and guide-vane setting. The absolute-pressure level of the test system is the independent variable, and this is decreased until changes are observed in the machine performance. For a turbine, these changes will appear in the flow rate, the power output, and the efficiency. In some laboratories, however, turbine cavitation tests are made by operating at different heads and speeds, but at the same unit head and unit speed. The results are then shown as changes in unit power, unit flow rate, and efficiency.

If the machine is a pump, cavitation tests can be made in two ways. One method is to keep the speed and suction head constant and to increase the discharge up to a cutoff value at which it will no longer pump. The preferable method is to maintain constant speed and flow rate and observe the effect of suction pressure on head, power (or torque), and efficiency as the suction pressure is lowered. In such cases, continual small adjustments in flow rate may be necessary to maintain it at constant value.

Figure 8.6 shows curves for a turbine, obtained by operating at constant head, speed, and gate. Figure 8.7 shows curves for a pump, obtained from tests at constant speed and flow rate. These curves are typical in that each performance characteristic shows little or no deviation from its normal value

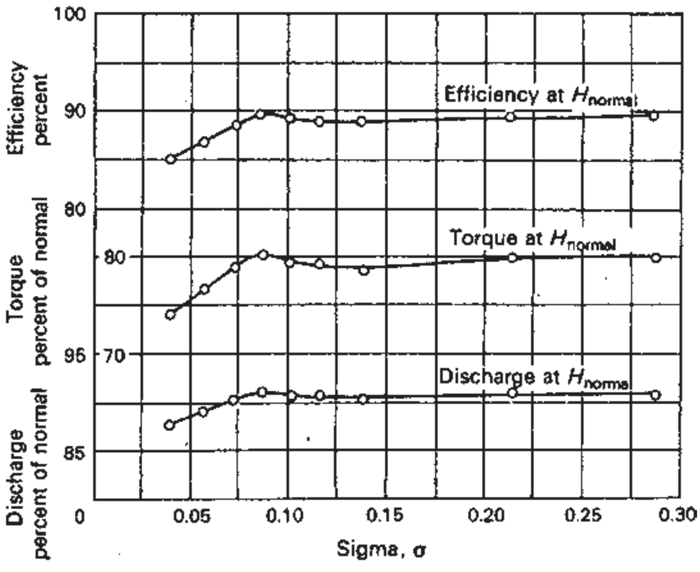

Figure 8.6 Sigma curves for a hydraulic turbine under constant head, speed, and gate opening. (Normal torque, head, and discharge are the values at best efficiency and high sigma.)

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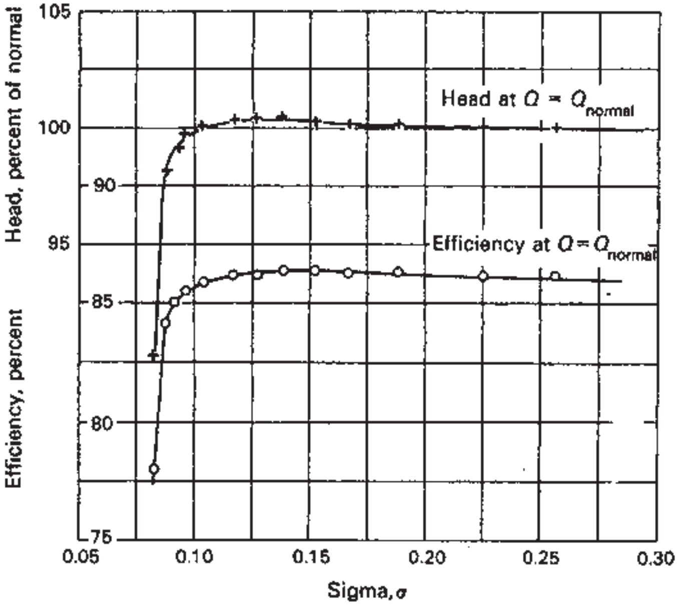

Figure 8.7 Sigma curves for a centrifugal pump at constant speed and discharge. (Normal head and discharge are the values at best efficiency and high sigma.)

(at high submergence) until low sigmas are reached. Then deviations appear, which may be gradual or abrupt.

In nearly all cases, the pressure head across a pump or turbine is so small in comparison with the bulk modulus of the liquid such that change in system pressure during a sigma test produces no measurable change in the density of the liquid. Thus, in principle, until inception is reached, all quantities should remain constant and the σ curves horizontal.

Figure 8.8 shows some of the experimental sigma curves obtained from tests of different pumps. It will be noted that the first deviation of head H observed for machines A and C is downward but that for machine B is upward. In each case, the total deviation is considerably in excess of the limits of accuracy of measurements. Furthermore, only machine A shows no sign of change in head until a sharp break is reached. The only acceptable conclusion is, therefore, that the inception point occurs at much higher value of sigma than might be assumed and the effects of cavitation on the performance develop very slowly until a certain degree of cavitation has been reached.

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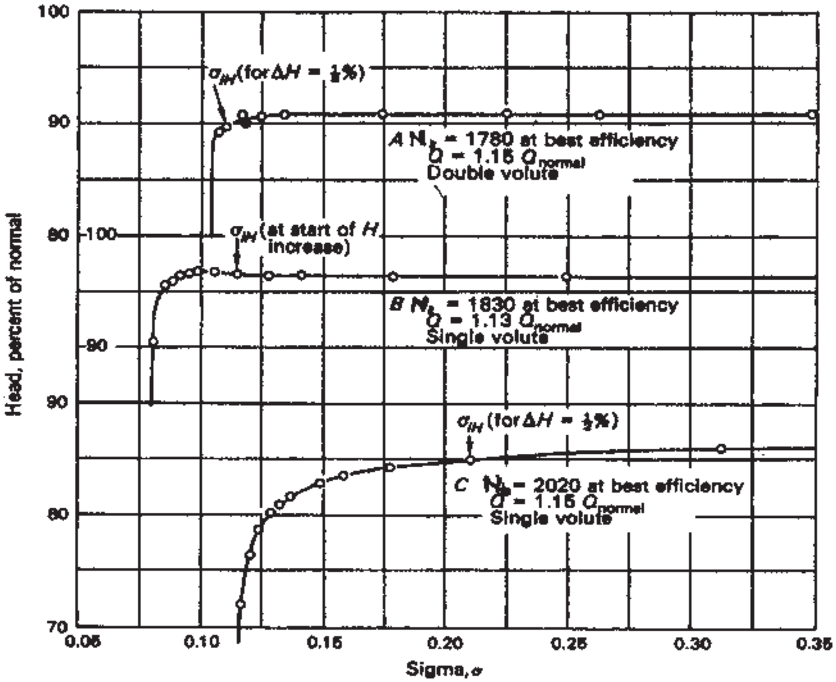

Figure 8.8 Comparison of sigma curves for different centrifugal pumps at constant speed and discharge. (Normal head and discharge are the values at best efficiency and high sigma.)

8.8.3 Interpretation of Sigma Tests

The sigma tests described are only one specialized use of the parameter. For example, as already noted, sigma may be used as a coordinate to plot the results of several different types of experience concerning the effect of cavitation of machines. Even though sigma tests are not reliable in indicating the actual inception of cavitation, attempts have often been made to use them for this purpose on the erroneous assumption that the first departure from the noncavitating value of any of the pertinent parameters marks the inception of cavitation. The result of this assumption has frequently been that serious cavitation damage has been observed in machines whose operation had always been limited to the horizontal portion of the sigma curve.

Considering strictly from the *effect* of cavitation *on the operating characteristics*, the point where the sigma curve departs from the horizontal may

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be designed as the inception of the effect. For convenience in operation, points could be designated as σ_{iP}, σ_i, σ_{iH}, or σ_{iQ}, which would indicate the values of σ_i for the specified performance characteristics. In Fig. 8.8, such points are marked in each curve. For pumps A and C, the indicated σ_{iH} is at the point where the head has decreased by 0.5% from its high sigma value. For pump B, σ_{iH} is shown where the head begins to increase from its high sigma value.

The curves of Fig. 8.8 show that at some lower limiting sigma, the curve of performance finally becomes nearly vertical. The knee of this curve, where the drop becomes very great, is called the breakdown point. There is remarkable similarity between these sigma curves and the lift curves of hydrofoil cascades. It is interesting to note that the knee of the curve for the cascade corresponds roughly to the development of a cavitation zone over about 10% of the length of the profile and the conditions for heavy vibrations do not generally develop until after the knee has been passed.

8.8.4 Suction Specific Speed

It is unfortunate that sigma varies not only with the conditions that affect cavitation but also with the specific speed of the unit. The suction specific speed represents an attempt to find a parameter, which is sensitive only to the factors that affect cavitation.

Specific speed as used for pumps is defined as

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}} \tag{8.34}$$

where N is the rotating speed, Q the volume rate of flow, and H the head differential produced by pump.

Suction specific speed is defined as

$$S = \frac{N\sqrt{Q}}{H_{sv}^{3/4}} \tag{8.35}$$

where H_{sv} is the total head above vapor at pump inlet. Runners in which cavitation depends only on the geometry and flow in the suction region will develop cavitation at the same value of S. Presumably, for changes in the outlet diameter and head produced by a Francis-type pump runner, the cavitation behavior would be characterized by S. The name “suction specific speed” follows from this concept. The parameter is widely used for pumping machinery but has not usually been applied to turbines. It should be equally applicable to pumps and turbines where cavitation depends only on the suction region of the runner. This is more likely to be the case in low-specific-speed Francis turbines. The following relation between specific speed (as used for pumps), suction specific speed, and

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sigma is obtained from Eqs. (8.34) and (8.35).

$$\frac{N_{s-\text{pump}}}{S} = \left(\frac{H_{sv}}{H}\right)^{3/4} = \sigma_{sv}^{3/4} \tag{8.36}$$

A corresponding relation between specific speed as used for turbines, suction specific speed, and sigma can be obtained from Eqs. (8.33) and (8.35) together with the expression

$$\text{hp} = \frac{\eta_t \gamma Q H}{550}$$

where η_t is the turbine efficiency.

Then

$$\frac{N_{s-\text{turb}}}{S} = \sigma_{sv}^{3/4}\left(\frac{\eta_t \gamma}{550}\right)^{1/2} \tag{8.37}$$

It is possible to obtain empirical evidence to show whether or not S actually possesses the desirable characteristic for which it was developed, i.e., to offer a cavitation parameter that varies only with the factors that affect the cavitation performance of hydraulic machines and is independent of other design characteristics such as total head and specific speed. For example, Fig. 8.9

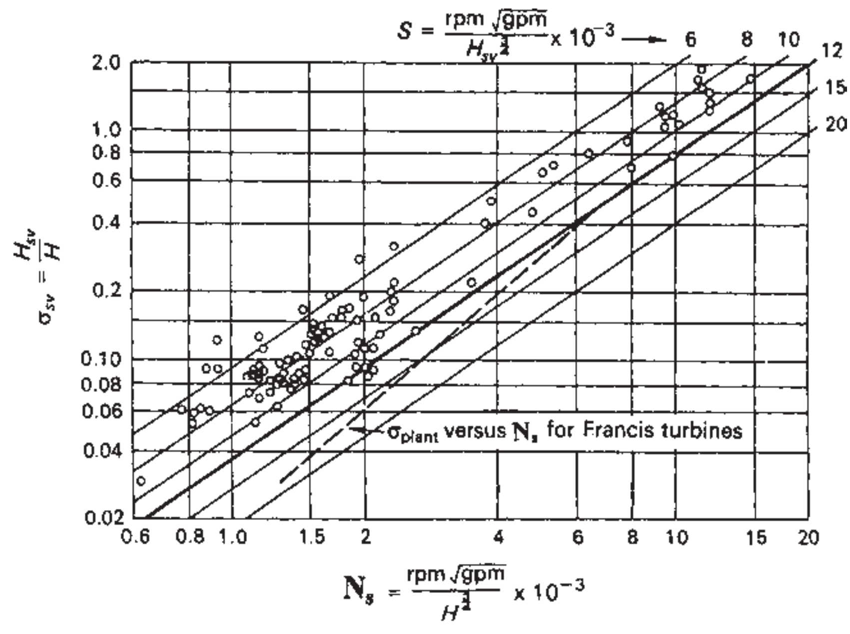

Figure 8.9 Sigma vs. specific speed for centrifugal, mixed-flow, and propeller pumps.

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shows a logarithmic diagram of sigma vs. specific speed on which are plotted points showing cavitation limits of individual centrifugal, mixed-flow, and propeller pumps. In the same diagram, straight lines of constant S are shown, each with a slope of $(\log \sigma_{sv})/(\log N_s) = 3/4$ [Eq. (8.36)]. It should be noted that σ_{sv} and S vary in the opposite direction as the severity of the cavitation condition changes, i.e., as the tendency to cavitate increases, σ_{sv} decreases, but S increases.

If it is assumed that as the type of machine and therefore the specific speed change, all the best designs represent an equally close approach to the ideal design to resist cavitation, then a curve passing through the lowest point for each given specific speed should be a curve of constant cavitation performance. Currently, the limit for essentially cavitation-free operation is approximately $S = 12{,}000$ for standard pumps in general industrial use. With special designs, pumps having critical S values in the range of 18,000–20,000 are fairly common. For cavitating inducers and other special services, cavitation is expected and allowed. In cases where the velocities are relatively low (such as condensate pumps), several satisfactory designs have been reported for S in the 20,000–35,000 range.

As was explained, Fig. 8.5 shows limits that can be expected for satisfactory performance of turbines. It is based on experience with installed units and presumably represents good average practice rather than the optimum. The line for Francis turbines has been added to Fig. 8.9 for comparison with pump experience. Note that allowable S values for turbines operating with little or no cavitation tend to be higher than those for pumps when compared at their respective design conditions. Note also that the trend of limiting sigma for turbines is at a steeper slope than the constant S lines. This difference of slope can be taken to indicate that either the parameter S is affected by factors other than those involved in cavitation performance or the different specific-speed designs are not equally close to the optimum as regards cavitation. The latter leads to the conclusion that it is easier to obtain a good design from the cavitation point of view for the lower specific speeds.

NOTATION

a	Acceleration
A	Area
C_p	Pressure coefficient
E	Modulus of elasticity
h	Elevation
H	Head
K	Cavitation parameter
KE	Kinetic energy
N	Rotation speed

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N_s	Specific speed
p	Pressure
Q	Flow rate
r	Radial distance
r_m	Mean radius
R	Radius of cavity wall
S	Suction specific speed
t	Time
u	Velocity
V	Velocity
Z	Dimensionless volume of bubble
ρ	Density
η_t	Turbine efficiency
γ	Specific weight
t'	Dimensionless time
σ	Cavitation parameter

SUFFIXES

0	Undisturbed fluid properties
atm	Atmospheric values
d	Dynamic effects
e	Exit
f	Friction
i	Inception properties
min	Minimum
r	Radial
s	Static
v	vapor

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